Decreasing max Lateness

We can prove this algorithm by using a using a few theories

1. The optimal algorithm has no idle time between any two jobs. It seems obvious to do so in order to decrease max lateness
2. The second one is the presence of exchange pairs. As our greedy algorithm is based on the fact of taking the job which has the earliest deadline in ascending order, in such a case an inversion pair is a pair such that – deadline of k is before j but is scheduled after j, don’t exist.
3. The third theory is that if there exists an inversion pair k, l then there exists an adjacent inversion pair to it too. (can be easily proved by taking a list)

We now prove the following. We have n optimal solutions and we choose the best solution such that it has the least amount of inversion pairs among all the solutions. Now using the above, the argument, if it were to have an inversion pair, then it would also have an adjacent inversion pair too.

We see if this pair to be inverted it wouldn’t change the max lateness of this problem. And now becomes a contradiction that there exists a solution with only these many inversions.

**Note - such an argument is called the Exchange argument. Gradually transform any solution to the one found by the greedy algorithm without hurting its quality.**